Mathematics: analysis and approaches
Higher Level
Paper 1

Name

Date: ____________________

2 hours

Instructions to candidates

- Write your name in the box above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your name on each answer sheet and attach them to this examination paper.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

exam: 12 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

Let $f(x)=\dfrac{1}{2x+1}$ and $g(x)=2x-3$. Given that $h(x)=(f\circ g)(x)$, find:

(a) $h(x)$; [2]

(b) $h^{-1}(x)$. [4]

2. [Maximum mark: 5]

The first derivative of a function g is given by $(x-4)^3$.

(a) Find the second derivative of g. [2]

(b) Write down the value of $g''(4)$. [1]

(c) The x-coordinate of point A on the graph of g is 4. Explain why A is **not** a point of inflexion. [2]

3. [Maximum mark: 7]

Given that $\log_3 2 = x$ and $\log_3 5 = y$, express each of the following in terms of x and y.

(a) $\log_3 20$ [2]

(b) $\log_3\left(7\tfrac{13}{16}\right)$ [2]

(c) $\log_5 8$ [3]

4. [Maximum mark: 6]

Consider the infinite series $1+\ln x+(\ln x)^2+\cdots$.

(a) Find the values of x such that the series converges. [3]

(b) Find the value of x such that the series converges to 2. [3]

5. [Maximum mark: 6]

The point $P(p, -1)$ lies on the curve $7y^3 + xy^2 - x^2y + 1 = 0$. Given that the gradient of the line tangent to the curve at P is $\frac{5}{18}$, find the value of p. [6]

6. [Maximum mark: 6]

Solve the equation $8\sin x\cos x=\sqrt{12}$, for $0\le x\le\frac{\pi}{2}$.

7. [Maximum mark: 6]

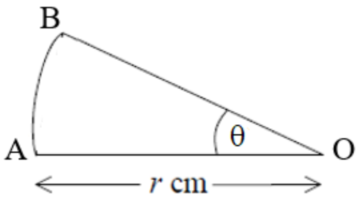
If α and β are the roots of the equation $2x^2 - x + 4 = 0$, find a quadratic equation with integer coefficients whose roots are $\alpha + 2$ and $\beta + 2$.

8. [Maximum mark: 7]

Prove by mathematical induction that $4^n + 2$ is a multiple of 3 for all positive integers. [7]

9. [Maximum mark: 6]

The figure shows a sector OAB of a circle of radius r cm and centre O, where $\mathrm{A\hat{O}B} = \theta$.



The value of r is increasing at the rate of 2 cm per second and the area of the sector is increasing at the rate of 2π cm^2 per second. At the moment when $r = 3$ cm and $\theta = \frac{\pi}{4}$, find the rate of increase of θ indicating the units for this rate of change.

Do **not** write solutions on this page.

Section B

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

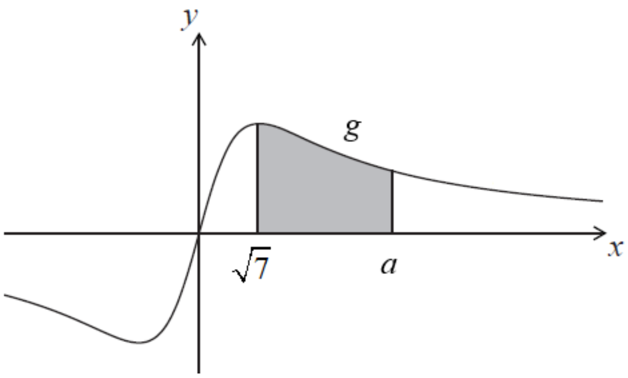
10. [Maximum mark: 16]

The function g is defined by $g\left(x\right)=\dfrac{3x}{x^2+7}$.

(a) Show that $g'\left(x\right)=\dfrac{21-3x^2}{\left(x^2+7\right)^2}$. [4]

(b) Find $\displaystyle\int\frac{3x}{x^2+7}\,\mathrm{d}x$. [5]

The diagram below shows a portion of the graph of g.



(c) The shaded region is enclosed by the graph of g, the x-axis, and the lines $x=\sqrt{7}$ and $x=a$. This region has an area of $\ln 8$. Find the value of a. [7]

Do **not** write solutions on this page.

11. [Maximum mark: 17]

Consider the complex number z such that $|z| = |z - 3\text{i}|$.

(a) Show that the imaginary part of z is $\dfrac{3}{2}$. [2]

(b) Let z_1 and z_2 be the two possible values of z, such that $|z| = 3$

(i) Sketch a diagram to show the points represent z_1 and z_2 in the complex plane, where z_1 is in the first quadrant. [2]

(ii) Show that $\arg z_1 = \dfrac{\pi}{6}$. [1]

(iii) Write down the value of $\arg z_2$. [1]

(c) Given that $\arg\left(\dfrac{z_1^k z_2}{2\text{i}}\right) = \pi$, find a value of k. [5]

(d) Find an expression for the sum of the first 20 terms of the series

$$\ln\left(x^2\right) + \ln\left(\frac{x^2}{y}\right) + \ln\left(\frac{x^2}{y^2}\right) + \ln\left(\frac{x^2}{y^3}\right) + \cdots$$

Giving your answer in the form $\ln\left(\dfrac{x^m}{y^n}\right)$ where m and n are positive integers. [6]

12. [Maximum mark: 22]

(a) Using an identity for $\cos 2\theta$, show that $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$. [2]

(b) Hence, find $\int \cos^2 x \, \text{d}x$ [4]

Functions f and g are defined such that $f(x) = 4\cos x$ and $g(x) = \sec x$ for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$.
Let R be the region enclosed by the two functions.

(c) Find the value of the x-coordinate for each of the two points of intersection of f and g. [4]

(d) Sketch the graphs of f and g and clearly shade the region R. [3]

The region R is rotated through 2π radians about the x-axis to generate a solid.

(e) (i) Write down a definite integral that represents the volume of the solid.

(ii) Hence, find the volume of the solid. [9]